On the Computational Power and Complexity of Spiking Neural Networks

Johan Kwisthout

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Joint work with Nils Donselaar and Arne Diehl

Donders Center for Cognition
Department of Human-Centred Intelligent Systems

Workshop on Theory of Neuromorphic Computing - June 2026

Overview

- Neuromorphic computing: interesting new approach
  - Intel Loihi, Manchester Spinnaker systems
  - Energy as resource on par with time and space
  - Potential for orders of magnitude more efficient
  - But hard to compare with Von Neumann architectures
- Can we have a neuromorphic complexity theory?
- Is this architecture really something new?
- Are the inherently energy-inefficient problems?
- Invitation for collaborations!

Agenda

- Introduction
  - Who am I?
  - Research group

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- Motivation - why this work?
  - Neuromorphic computing
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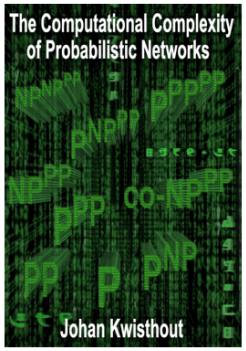
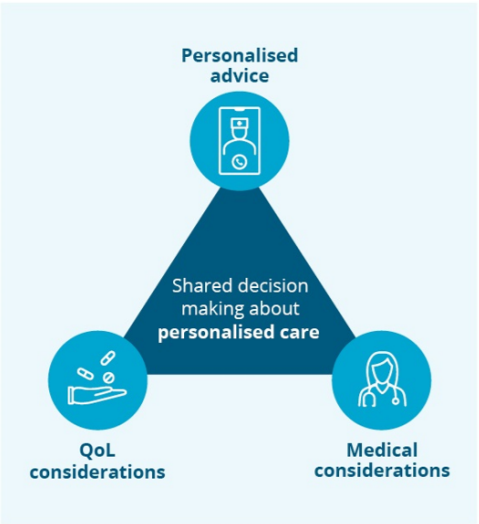
- Structural complexity results

Who am I



PersOn | Personalised Care in Oncology



ELSA lab
for Decision Support

P G M
G R O U P

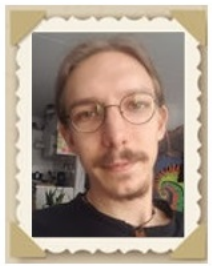
<AI>RADBOUD UNIVERSITY</AI>
Artificial Intelligence Program

Research Group

Research Group

- PGMs in Decision Support Systems
  - Explainability, trustworthiness, accountability, QoL, online maintenance, traceability
  - Foundations: Credal networks, Learning, Model checking
  - Applications: Healthcare, AutoML, Sustainability
- Foundations of stochastic computing
  - Parameterized complexity, approximate inference
  - Stochastic computing, synchronized stochasticity
  - Neuromorphic computing




Intel INRC

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- I have been part of (and modestly supported by) Intel's Neuromorphic Research Community from its start in 2019
- Work presented today stems from this period
- Our university was the second programme in the world (Carnegie Mellon being the first one) that offered students (remote) access to Intel's Loihi chip ...
- ... and we were among the first that offered a graduate course on neuromorphic computing
- However, I am currently not very active in the field anymore

Paper

- Most of today's presentation comes from this paper: `https://iopscience.iop.org/article/10.1088/2634-4386/ae2cc1`

Neuromorph. Comput. Eng. **6** (2026) 014004 https://doi.org/10.1088/2634-4386/ae2cc1

NEUROMORPHIC
Computing and Engineering

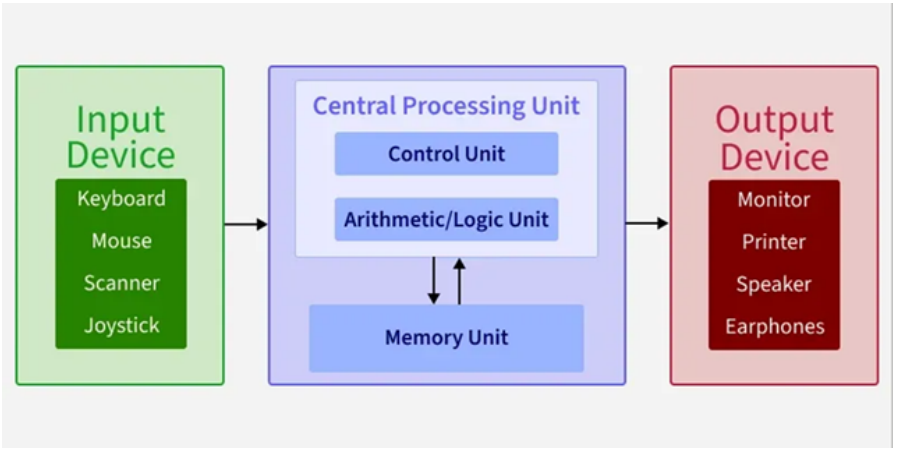
PAPER

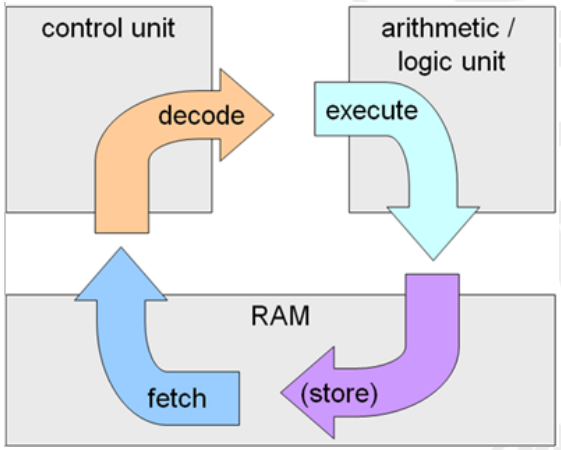
Neuromorphic complexity theory: computational models and complexity classes for spiking neural networks*

Johan Kwisthout[1], Arne Diehl[**] and Nils Donselaar[2]

Von Neumann architecture

- Let's start describing the Von Neumann architecture and processing cycle, just to explain what is different in neuromorphic computing





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  - In this talk: architectures where data and algorithm are co-located in spiking neural network architectures
- What can we do with these architectures?
- What can they do better than CPUs / GPUs?

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  - **Graphics** Processing Unit
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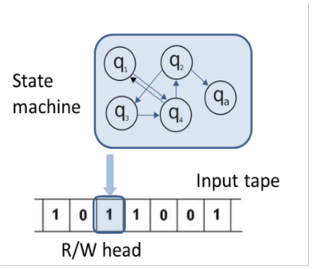
- Nobody thought of deep learning as useful application of GPUs prior to AlexNet crushed the competition in 2012...
  - Now 90% of GPU energy use is in AI...
- Lesson learnt: Maybe there are other tasks that can be sped up with a neuromorphic chip besides machine learning?
- For example, scientific computing, graph-like problems

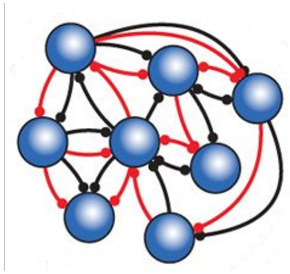
Example problems with neuromorphic algorithms

- Finding shortest paths (Dijkstra's algorithm)
- Finding minimum spanning trees
- Finding number of dominating sets
- Longest increasing subsequence
- We worked on several of these problems and built a GUI / simulator for hand-crafting networks: `https://github.com/nielsvharten/SNN-builder/`

Spiking Neural Network (SNN)

- Spiking Neural Network model as computational model for neuromorphic architectures



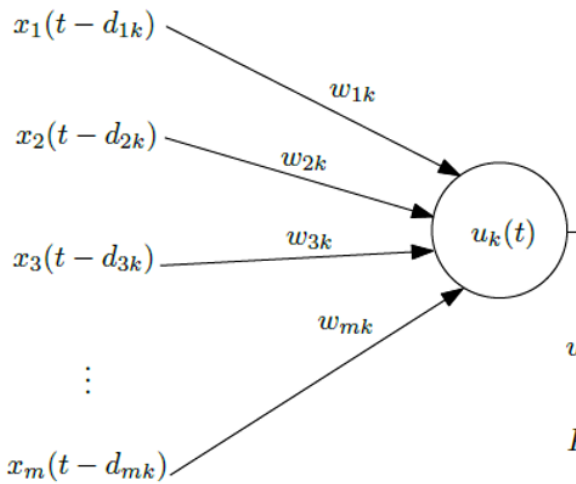


- Deterministic neurons
- Specific I/O neurons
- Discrete time steps

- Key neuromorphic aspects are there:
  - Co-located memory & computation
  - Spiking behaviour $\rightarrow$ energy efficiency
  - Underlying principle of Loihi / SpiNNaker machines
- Abstraction – some properties are lost

SNN definitions

- We have programmed input neurons, dedicated readout neurons, & regular LIF neurons with potentials, threshold and reset voltages, and leakage; synapses with delays and weights

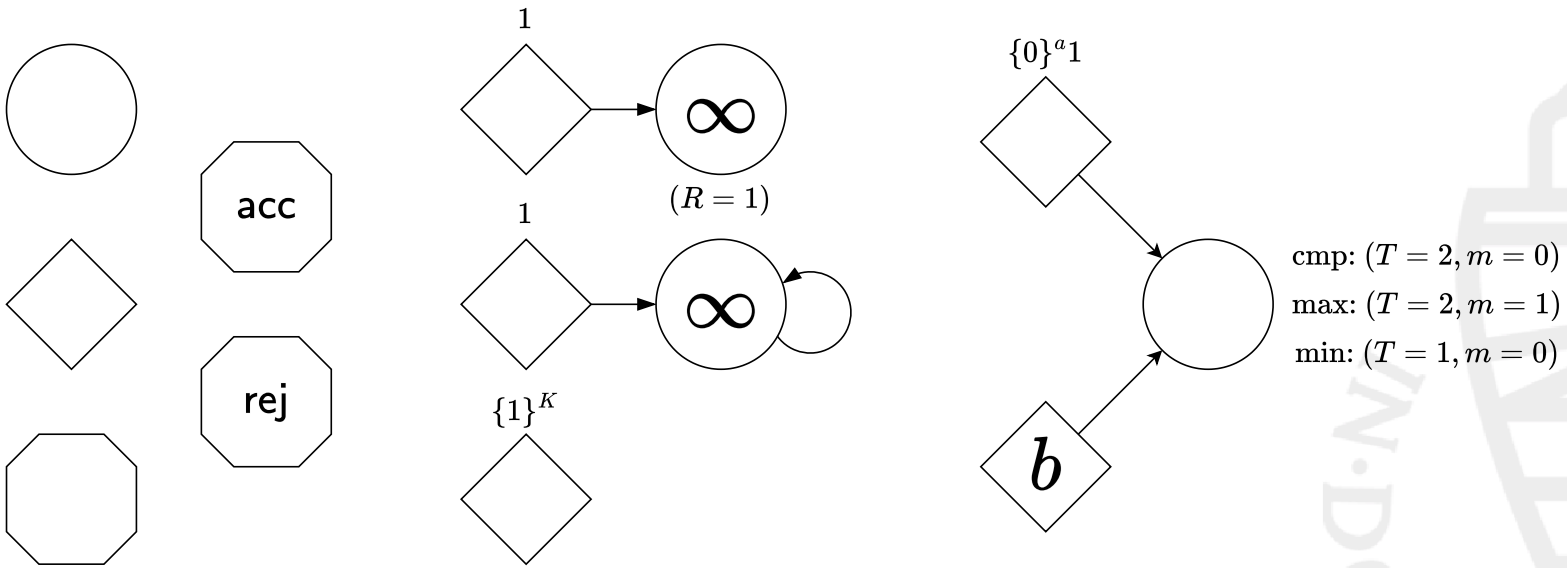


$$u_k(t) = \begin{cases} \max(0, R_k + \sum_j w_{jk} x_j(t-d_{jk})) & \text{if } u_k(t-1) \geq T_k; \\ \max(0, m_k u_k(t-1) + \sum_j w_{jk} x_j(t-d_{jk})) & \text{otherwise.} \end{cases}$$

$R_k = \text{reset voltage}, T_k = \text{threshold}, m_k = \text{leakage constant}$

$$x_k(t) = \begin{cases} 1 & \text{if } u_k(t) \geq T_k; \\ 0 & \text{otherwise.} \end{cases}$$

Notation and simple circuits



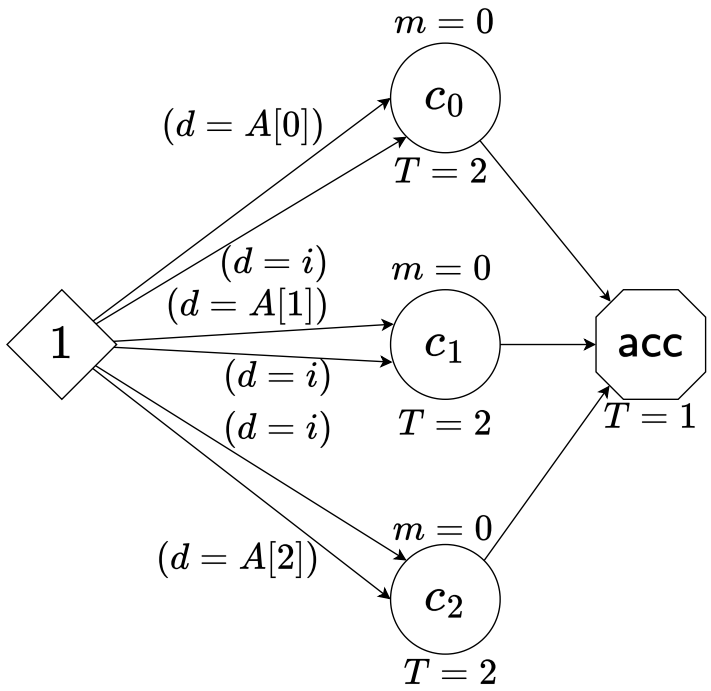
Left: regular, input, and readout neuron; dedicated accept, reject neuron

Mid: three ways of encoding a constantly firing neuron

Right: comparing a natural number against a reference value

Solving a simple problem: take 1

Decide whether an array A of natural numbers contains i.

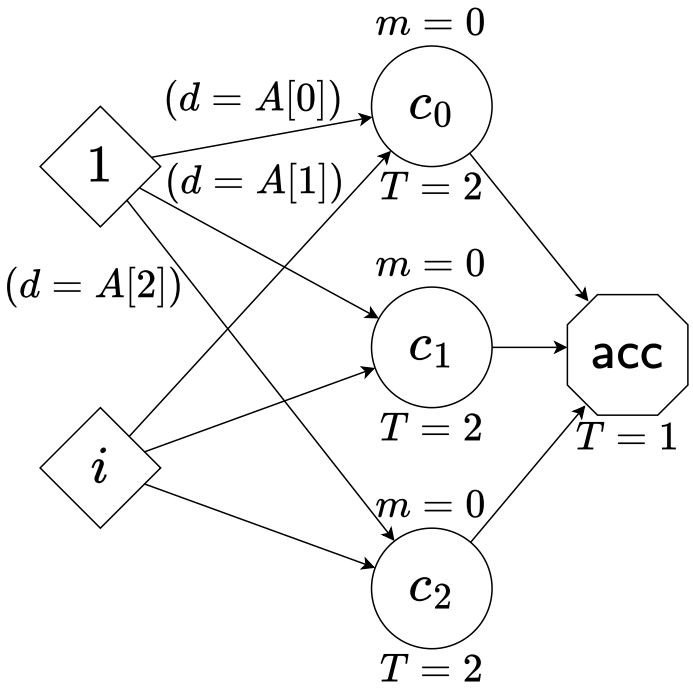


Concept: both A and i (the data) as well as the parallel comparison (the computation) are encoded in the machine

Energy: $\leq |n| + 2$ spikes needed

Solving a simple problem: take 2

Decide whether an array A of natural numbers contains i.

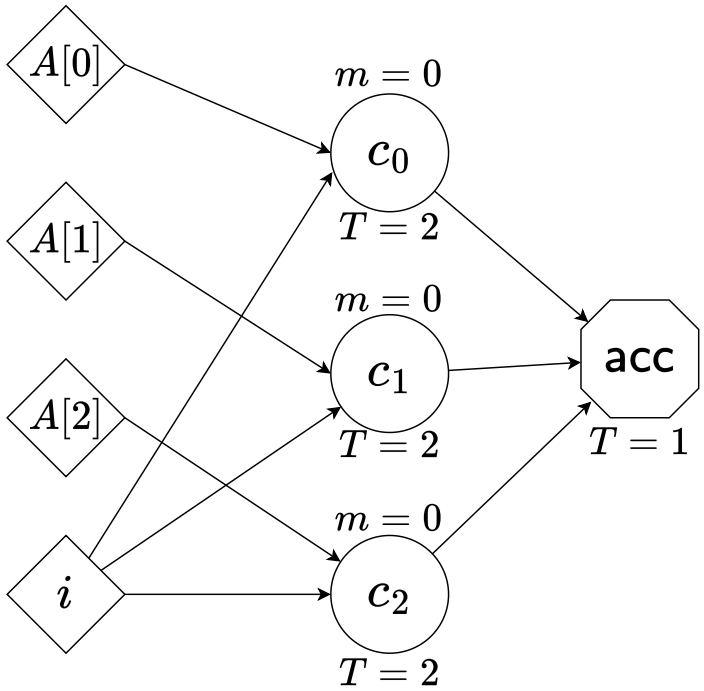


Concept: i is now offered as input using a single spike with delay i; A and the computation are encoded in the machine

Energy: $\leq |n| + 3$ spikes needed

Solving a simple problem: take 3

Decide whether an array A of natural numbers contains i.



Concept: both the search value and the integers in the array are offered as input to the machine, while the size of the array is fixed

Energy: $\leq 2|n| + 2$ spikes needed

Generality and resources of SNN computations

- Observation 1: we can do *program* an SNN to do basic computations such as finding an integer in an array

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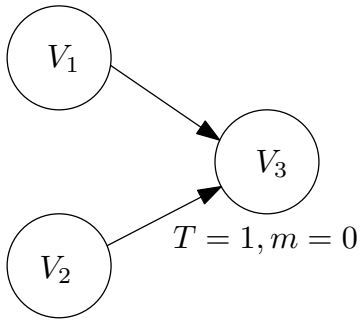
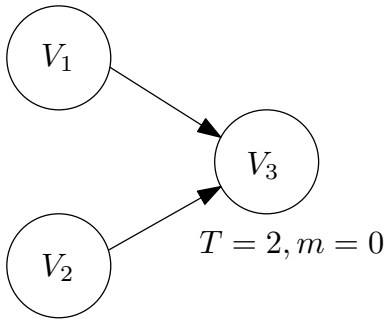
- Observation 2: computation and data can be co-located in the network and become morphed
- Observation 3: timing (when does a spike arrive) using delays is crucial in the computation
- Observation 4: there is a trade-off between generality of the network and resources needed for the computation
- Observation 5: the final example looks quite like a Boolean circuit family, with different circuits for each input size - can we do more generic computations (rather than non-uniform)?

A family of SNNs simulates a Boolean circuit family

- We can simulate binary operations with simple circuit components when we consider a spike at a particular moment to indicate a 1 and the absence of a spike as a 0:



$V_3 = V_1 \vee V_2$



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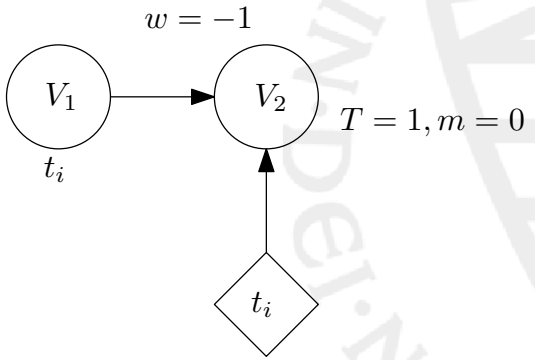
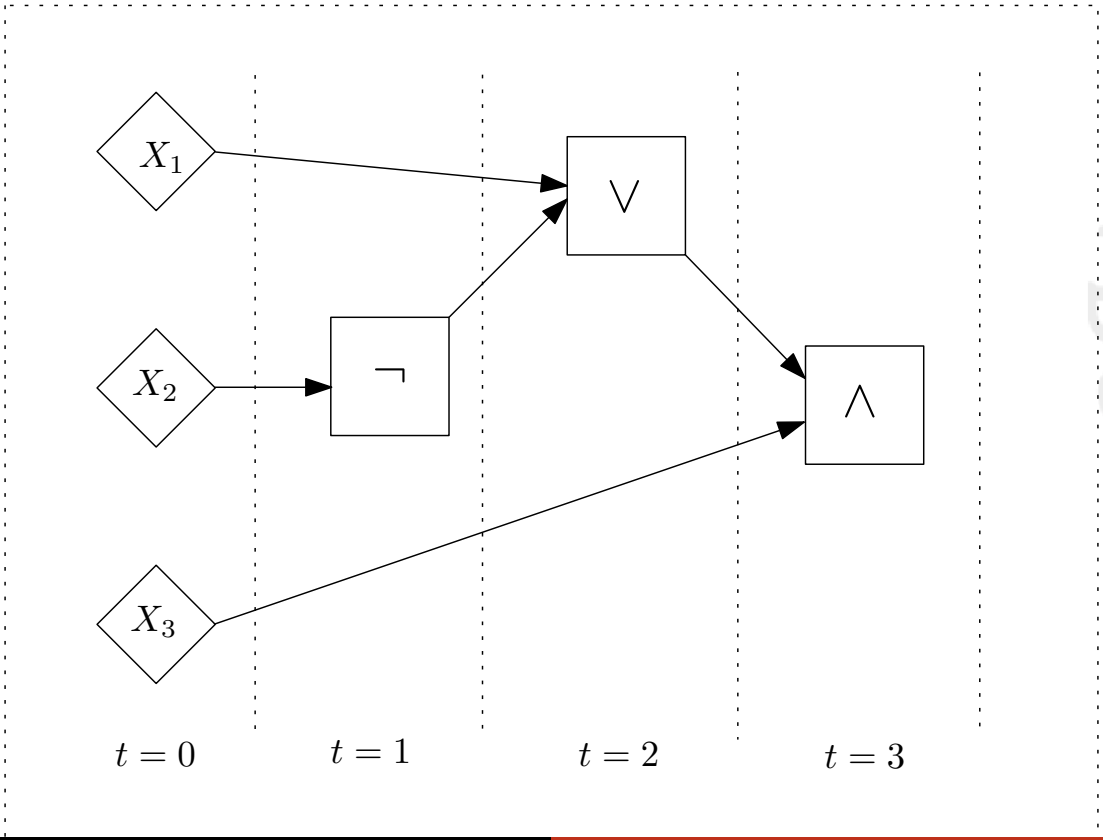


$V_2 = \neg V_1$

A family of SNNs simulates a Boolean circuit family

- Simulating $(x_1 \vee \neg x_2) \wedge x_3$ with input spikes (or absence of it) at $t = 0$ using these components. Spikes need to be delayed to arrive at the appropriate time step at the 'gate'.



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- Likely, for bounded precision and no plasticity, an SNN has the same complexity as a finite state automaton

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- e.g. structural: is there something like an energy hierarchy?
- computational: are there problems that are complete for some complexity class?
- We need to build a framework: decide on a machine model, identify resources and hierarchies of complexity classes
- We will start with studying some formal properties of resources and defining different machine models!

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- Note these resource definitions are all abstractions from the physical world (e.g., no spike $\neq$ no energy)
- We can prove some general properties about these resources

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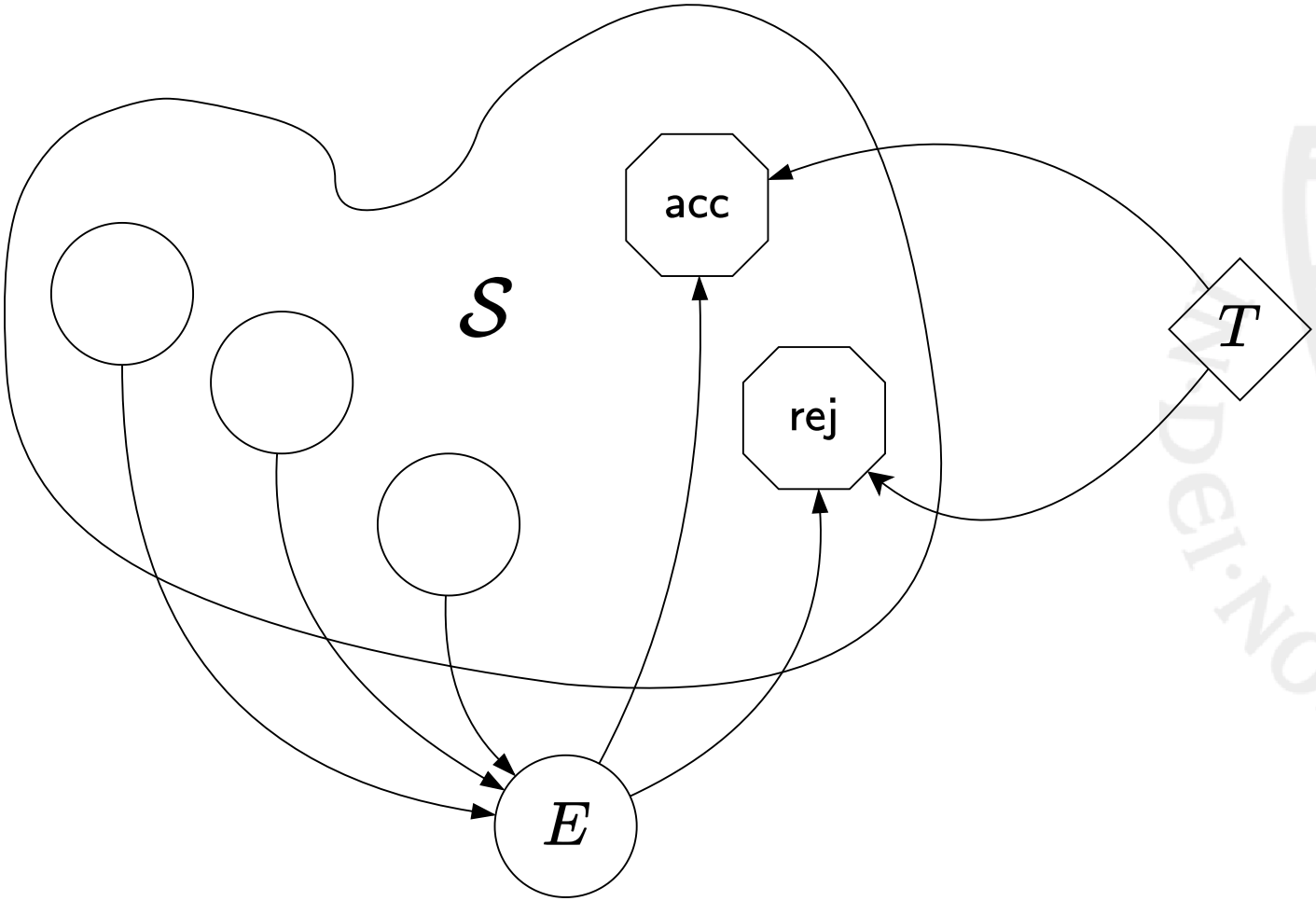
- We use dedicated accept and reject neurons; a computation starts at $t = 0$ at the first instance of the spike train and ends when either neuron fires
- We can limit computation time and energy usage (and go in reject mode if exceeded) using a **clock** and a **meter**

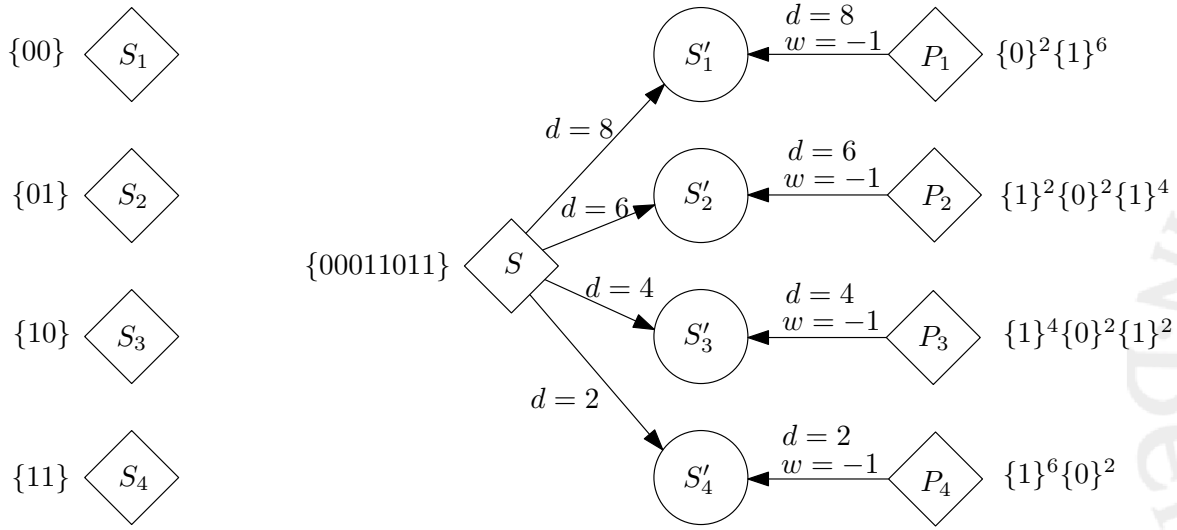
Clock and meter

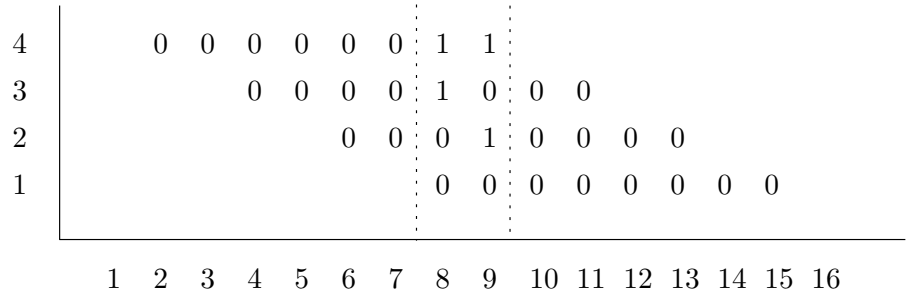
- Adding a clock and a meter to constrain computations



Generalisation: spike wave

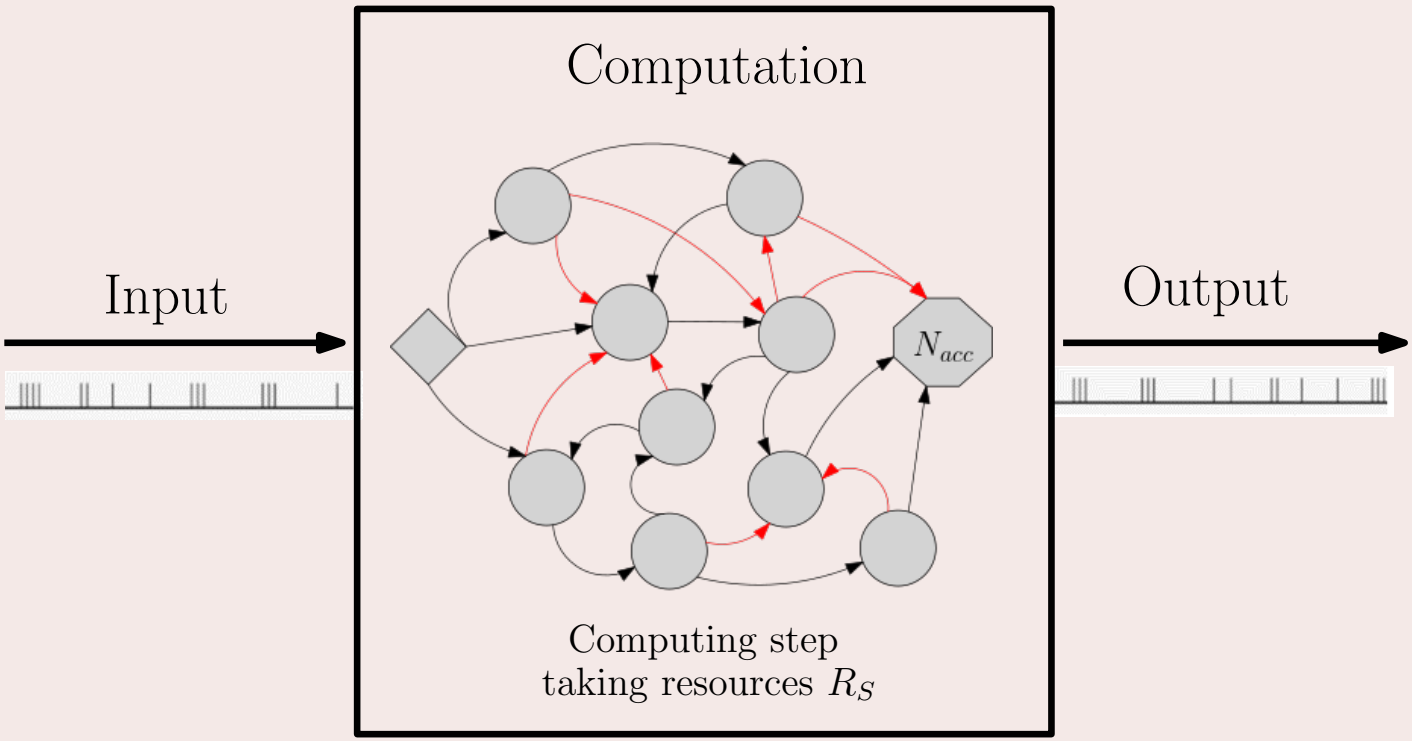
- Some generalisation: a spike wave, rather than a single input, can be simulated by a single stream





Machine models (1)

Spike-input model



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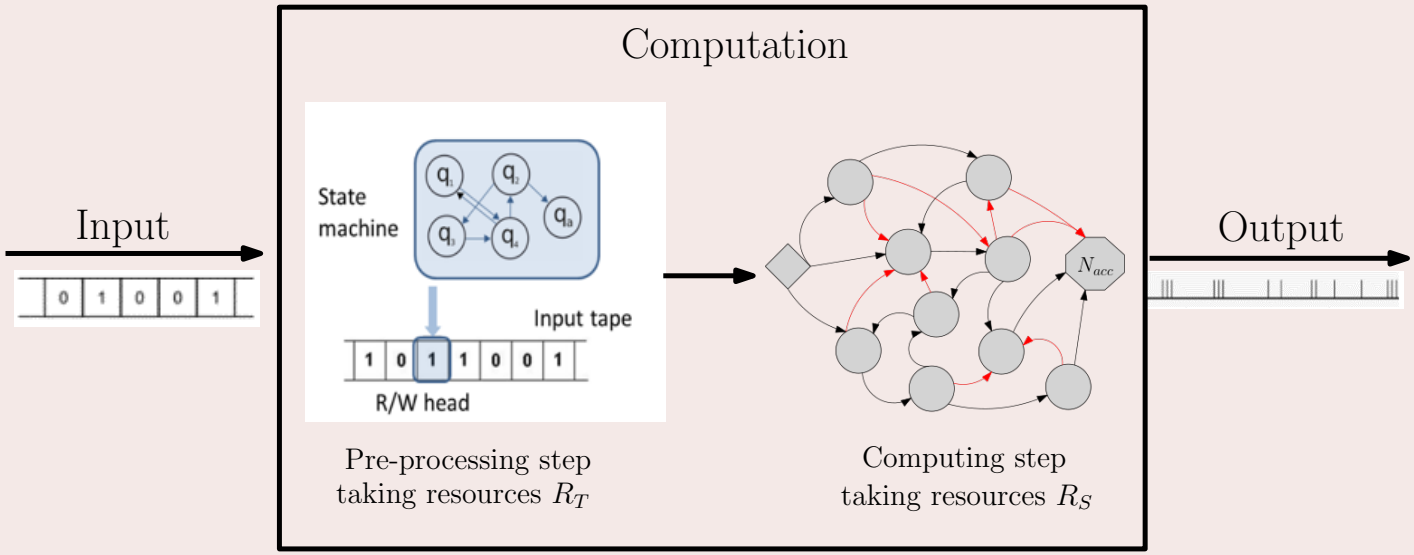
- It could compute, for example, whether an array of a given (fixed) size contains an number, where the numbers are given as input and accepts if it is, and rejects if it is not
- This is quite similar to a Boolean circuit
- A *family* of SNNs can be powerful, but as a singleton machine it is (given finite precision) effectively a finite state automaton
- It also does not really combine data and algorithm in one model and as such, does not harnass the power of SNNs

Machine models (2)

Pre-processing model



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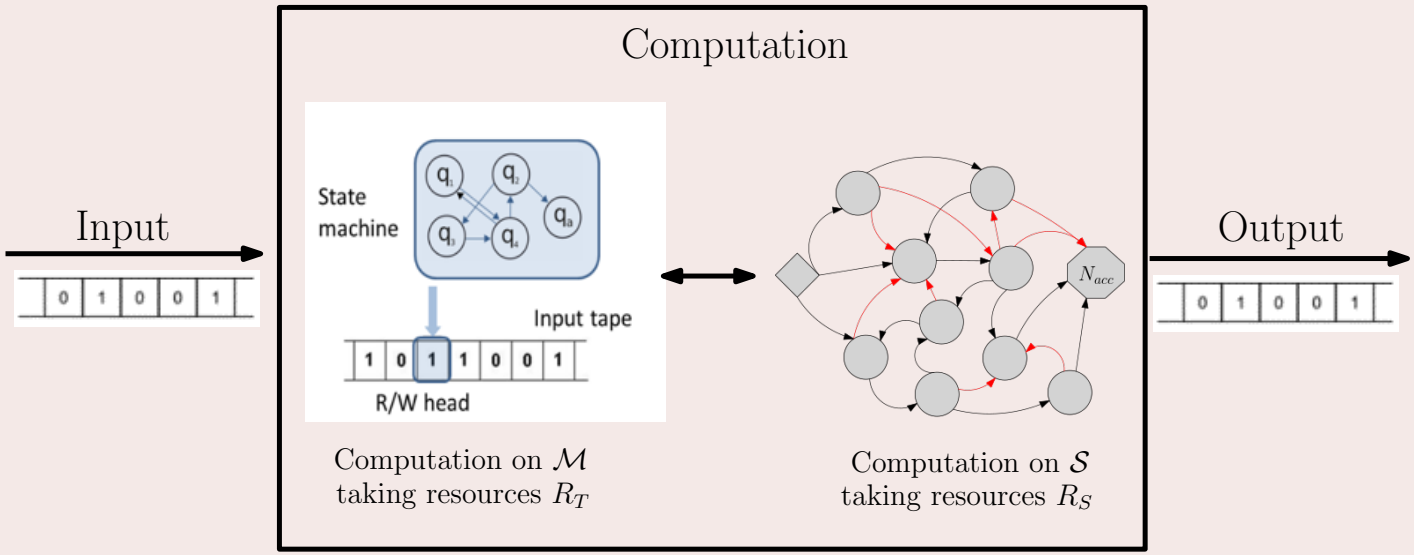
- This models a traditional algorithm, on input, constructing an SNN and delegating the computation to this machine
- It can be considered to be an extension of the advice function in non-uniform complexity theory where we map $I = \{1\}^n$ to a circuit that can deal with inputs of size n
- Here, we relax this constraint and can map any input $I = \{0, 1\}^n$ to a spiking neural network
- With polynomial resources for time and space for the mapping as well as for the SNN, this can be shown to be equal to P

Machine models (3)

Interactive model



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- This model is effectively a neuromorphic co-processor idea where we can, at any time, construct a network and offload some computation
- For example, computing shortest paths as sub-routine of computing a maximum network flow (MSc thesis project Abdullah Ali, 2019)
- Interesting idea, but..
  - We don't actually have such co-processors
  - We need to construct each network from scratch (the ability to modify an existing network has not been considered yet)
  - In practice this will be a massive overhead and really different from an FPU / GPU operating in the same memory space

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No magic in neuromorphic computing..

- Conjecture: for fixed precision, spike-input machine can be simulated by a finite state automaton as we have no memory or stack
- Proven in the paper: for polynomially constrained resources for the SNN, both the interactive and the pre-processing classes equal P
- More interesting: look into constrained resources for either the TM or the SNN
- For example, we can show that the P-complete Network Flow problem can be solved in Logspace with access to an SNN with linear resources

..but for some instances it may help!

- Brad Aimone and colleagues (2020) showed that finding a shortest path can be more efficient on a neuromorphic device than on a regular TM given constraints on a) the edge weights (logarithmic in number of nodes) and b) the number of edges in the shortest path (square root of number of edges)

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- Basically code edge weights as delays, inhibit neuron after the first incoming spike, and keep track of the path
- For some problem instances, the inherent parallelism and time-dependent nature of the network works very well!
- But, you can construct adversarial instances where the delays become the bottleneck

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Completeness results

- So far, only completeness for halting questions ("given an input, does the network accept the input within given resource bounds")
- Completeness for time does not really make sense - but most interesting part would be completeness for energy!
- Can we find an inherently energy-hungry problem?
- .. and could we find differences in energy needs in SNN and TM models?
- Conjecture: existence of energy hierarchy (with more spikes, strictly more can be computed)

Conclusion

- What do we have: different machine models and different complexity classes / hierarchies based on them
- Some basic first results and constructs
- No real breakthroughs, unfortunately..
- .. due to lack of resources (grants) and small community
- What will be the future of neuromorphic computing? So far, no killer application, little commercial interest, does not yet *really* live up to expectations!